

EFFECTS OF COMPOUND CIRCULAR MOTION.

At the Marquess of Northampton's *soirée*, on Saturday, 21st of March, 1846, amongst the several objects of interest exhibited, and in some instances explained, to the numerous visitors, the following chiefly attracted our attention :—Mr. Perigal's instrument to develop a peculiar law of compound motion, generating retrogressive or recurrent curves ; this, however, for its novelty and apparent great importance, we have treated in a separate article under this same division of our sheet.

NEW LAW OF COMPOUND MOTION.

By the instrument, mentioned in the above notice, Mr. Perigal developed a peculiar law of compound circular motion, “generating *retrogressive* or *recurrent* curves, by which the moving body, when it has reached the extreme points of the curve, retrogrades or returns back in exactly the same line along which it advanced ; constantly moving forward and backward from one extremity to the other ; and always tracing and retracing the same line as it alternately advances and recedes.”

These curves were produced by a complicated system of wheel-work, which Mr. Perigal stated to be capable of generating numberless varieties of curves dependent upon the ratio of the velocities of the movements ; but the instrument was, on this occasion, adjusted for the production of parabolas or hyperbolas, and a sort of figure of 8 curve resembling a lemniscate, which he shewed to be different forms of one and the same curve, just as circles and right lines are (limits or) varieties of ellipses. When the tracing-point passed through the center, the curve was at one of its extremes or limits ; and the two ovals of the figure of 8 were opened to their fullest extent of roundness ; but alterations in the angular adjustment of the movements caused these ovals to become more and more flattened, till they ultimately converged or collapsed, and be-

came in appearance a single line, terminating in points, with the form of a parabola or hyperbola; and the tracing-point travelled forward and backward from one extremity of the curve to the other repeatedly, without in the least degree thickening the line in one part more than in another; evidencing that it advanced and receded in exactly the very same path! In fact, the line of return might be considered as superposed upon the line of advance; as Mr. Perigal remarked, in reply to a very eminent mathematician, who objected that the figure of 8 curve was a line of the fourth order, inasmuch as it could be cut by a straight line in four places, while the parabola was a line of the second order, because it could be so cut only in two places; but the suggestion of its being a double line superposed seems to remove the difficulty.

Mr. Perigal mentioned that one of his objects was to exhibit the PARABOLA in the novel character of a *retrogressive* or recurrent curve of definite range; whence he inferred, that if a comet moved in that curve (as most of them are said to do) it might return after it had performed its allotted journey, and continue to visit us periodically; as several do, which are therefore supposed to travel in very elongated ellipses, although their apparent path more resembles a parabola.

We do not take upon ourselves to decide that this "*retrogressive*" curve of Mr. Perigal is or is not identical with the parabola of Apollonius; but we can bear testimony to its striking resemblance to the conic section; affording at least *prima-facie* evidence of its relationship, however much its newly discovered property of "*periodicity*" may shock the prejudices of those who have hitherto thought themselves learned in such matters. Besides, it is well known that reciprocating straight lines can be produced by combinations of circular motions; and, therefore, we cannot see why it should have been deemed impossible so to produce parabolas and hyperbolas, which, being curved lines, would even appear the less unlikely. Of the importance of the discovery in reference to the cometary theory, our scientific readers will judge for themselves; and such of them as are mathematicians can, for their own satisfaction, put the question to the proof by submitting the problem to analytical investigation.—*Literary Gazette*, 28th March, 1846.



At the next subsequent *soirée* (4th of April), was exhibited another new instrument of Mr. Perigal's, for the practical development of

ANOTHER NEW LAW OF COMPOUND MOTION,

proving that the parabola occurs as a *branch*, or *component part*, of a great variety of curves. This he showed by a modification of Professor Wheatstone's photometer; the curve selected being a trochoidal curve of eight symmetrical loops, which became eight cusped branches, or eight hyperbolic, or eight parabolic branches, &c. varying according to the radial adjustment of the bright steel ball representing a comet or planet; which, in this case, always moved in one and the same direction, instead of alternating in opposite directions, as in the experiment exhibited at the previous *soirée*. Mr. Perigal explained that, besides multitudes of other kinds, there are innumerable varieties of this particular kind of curve, having any imaginable number of symmetrical branches, approximating more or less nearly to the ellipse and parabola; in other words, that these are curves of which the ellipse, the hyperbola, and parabola, are limits or particular cases.

It seems to result, then, from these experiments, that it is not impossible for comets to *return periodically*, either forward and backward alternately, or always in the same direction, although moving in "curves which cannot be distinguished from parabolas;" quite as possible, indeed, as if they moved in elongated ellipses. (The consideration of the forces requisite to maintain them in such orbits is of course a totally distinct question.) That is to say there is nothing in the nature or properties of the curve itself to render the comet's periodical return impossible, now that the attribute of infinity, ascribed to the curve, is no longer admissible, except to express only the extreme or limit of the curve; just as a straight line may be imagined capable of infinite extension, but nevertheless may be drawn of any requisite degree of shortness, and is still a right-line whether short or long.—*Lit. Gazette*, 7th April, 1846.

Mr. Henry Perigal, literally, as well as figuratively, made a considerable noise among the learned with a piece of clock-work, which kept a small steel ball, representing a comet, in very rapid motion, in a curve of a very curious and beautiful figure—a sort of eight-rayed star, of which the eight branches were of a parabolic form. This, we understood, was intended to prove that a comet might return along the same path, and continue to visit us periodi-

cally, although moving "in a curve which cannot be distinguished from a parabola." This experiment differed from one he exhibited at the previous *soirée*, inasmuch as the curve then shown had but one branch instead of eight; and the comet moved alternately in opposite directions, backwards and forwards (always in exactly the same line), while in the present case the comet continued to move always in one and the same direction. These discoveries of Mr. Perigal are especially interesting and important at the present time, when so many as five comets have been visible at once in the starry firmament.—*Morning Post*, 6th April, 1846.

ON THE REVOLUTION AND ROTATION OF THE MOON.

At the *soirée* held at the house of the Marquess of Northampton, President of the Royal Society, on the 13th of February, 1847, Mr. H. Perigal exhibited and explained a variety of experiments, with a view to elucidate the theory of Revolution and Rotation, especially referring to the (hypothetical) motions that have been attributed to the Moon, to account for her presenting towards us always the same face or hemisphere. One of his instruments imparted to three ivory balls movements similar to those ascribed to the Moon in each of the three following hypotheses: and the resultant paths, which the component atoms of the Moon would in each case describe in space, were made evident to the eye by another little instrument which he called a Kinescope. With the aid of maps, on which were accurately laid down the positions of the Moon in accordance with the several hypotheses, these instruments and other auxiliar contrivances afforded ocular demonstration that, of the three, the last is the only one that will bear investigation.

1st hypothesis. That the Moon revolves round the Earth, and rotates on her own axis, in the same period, and in the same direction. These two circular motions would cause every individual atom of the Moon to move round the earth in a separate curve of the cardioid kind: these curves would vary in phase and dimensions according to the relative distance of each atom from the axis of rotation, and they would intersect in all directions.—In this case, the Moon, every time she travelled round the earth, would have two alternations of sidereal day and night, but only one alternation of terrestrial day and night; because, as regards the Stars which are outside her orbit, she would *turn round* twice, once round the Earth, and once round her own axis: but, as regards the Earth, which is inside her orbit, she would turn round only once; inasmuch as her orbital motion is a movement of migration or

translation, which alters her *bearing*, but not her *aspect* to the Earth round which she revolves.—Consequently, if the Moon were, simultaneously, to revolve around the Earth and to rotate about an axis within herself, with equal angular velocities in the same direction, all parts of her surface must be presented successively towards the Earth in the course of each rotation ; as the inevitable result of such double motion. Map 1.

2nd. That the Moon revolves round the Earth, and rotates on her own axis, in the same period, but in contrary directions.—These two circular motions would cause all the component atoms of the Moon to describe in space separate circles, all of the same size, intersecting in every direction.—In this case the Moon, each time she revolved round the Earth, would have one alternation of terrestrial day and night, for the same reason as before explained ; but no alternation of sidereal day and night : because, as regards the Stars outside her orbit, the turn she makes in one direction round her own axis counteracts the other turn she makes round the Earth. If the two axes were coincident, the two motions would destroy each other, and apparent rest would ensue ; but in this instance they would neutralize each other so far as this particular effect of turning round : consequently she would maintain what is called the parallelism of her diameters, a result which, hitherto, has been attributed erroneously to the action of one circular motion (revolution) alone.—Consequently, if the Moon were, simultaneously, to revolve around the Earth and to rotate about an axis within herself, with equal angular velocities in contrary directions, all parts of her surface must be presented successively towards the Earth in the course of each rotation ; as the inevitable result of such double motion. Map 2.

3rd. That the Moon revolves round the Earth, but does not rotate on her own axis.—This single circular motion would cause all the component atoms of the Moon to describe round the Earth concentric circles varying in size according to their radial distance from the centre of revolution.—In this case the Moon would have one sidereal day and night every time she revolved about the Earth ; because she would thereby turn round as regards the Stars ; while her *aspect* to the Earth being unchanged by her orbital motion, she would have no alternation of terrestrial day and night.—Consequently, if the Moon were to revolve about the Earth, without any additional rotation on an axis within herself, the same hemisphere must be presented constantly towards the Earth, as the inevitable

result of such single motion; and we should see the same face invariably, as we do now. Map 3.

Thus it was rendered evident to the eye, as well as to the mind, that the *last* hypothesis *may be true*; but that *neither* of the others would produce the effect that, hitherto, has been attributed to them by their respective advocates.

Wherefore, if the Moon orbitates about the Earth as its Satellite, any additional rotation in the same or in the contrary direction around her own centre of gravity is physically impossible; if it be true that she always presents towards us the same hemisphere, a fact which there seems no reason to doubt. Or, if she rotates on her own axis, she cannot also revolve about the Earth as her only centre of orbitation.—She cannot partake of *both* the circular movements in question, be they termed revolution and rotation, or distinguished by any other appellations: but, if our Satellite, she must orbitate around the Earth without any rotation on her own centre of gravity or about any axis within herself.—Consequently her *libration* is not caused, as asserted, by the inequality of two such movements; and remains to be accounted for.—If the Earth were *her* Satellite, she *might* rotate on her axis in the same period and in the same direction as the Earth revolved round her.—See *Literary Gazette*, 20th Feb. 1847.

At the Marquis of Northampton's *soirée*, 13th of March, 1847, Mr. Perigal exhibited several of his kinesiopes, producing spirallites and convolutes, certain curves which “progress” in spires (coils, or circumvolutions), so that any portion of any such curve is a portion of a spiral. Mr. Perigal explained that some of these curves were “interminate,” or capable of unlimited extension, such as the spiral of Archimedes, &c.; others finite, “returning into themselves” at the termination of their respective cycles, to reiterate the same path in the same direction; each forming (when completed) a continuous line, of which all the component parts are spiral. Of these finite curves there are, according to Mr. Perigal's account, innumerable varieties, distinguished by the number of their spires, and of their loops; the most simple being produced by the combination of two circular motions, of which the curves he exhibited were examples. Whence, Mr. Perigal asserts, the property of infinite extension, hitherto ascribed to all spirals, is applicable to comparatively a few kinds only; a vast majority of *spirals being parts of curves, which curves (if completed) return into themselves*.—*Literary Gazette*, 20th March, 1847.

At the Meeting of the Society of Arts, on 26th March, 1851, after the reading of Mr. Jopling's Paper on Geometric Curves, &c., MR. HENRY PERIGAL, jun., F.R.A.S., referring to the beautiful SPIRALS exhibited by Mr. Jopling and by Mr. Penrose, remarked that the *Reciprocals* of all BICIRCLOIDS (the *resultants* of two circular movements) are SPIRALS of a peculiar kind which *return into themselves*; the number of their *spires* or *circumvolutions* being in accordance with the relative velocities of the *epicycle* and *deferent*—the spiral character being most apparent when the velocity of the latter movement is much greater than that of the *epicyclic* movement. This he exemplified by some of his *Kinescopes*: instruments constructed after the principle of Wheatstone's *Photometer*, which imparted the required double motion to bright steel balls; so that the orbit or path of the ball in space was made evident to the eye (whence the term *kinescope*) by the light reflected from it in its rapid career. After exhibiting several of these "philosophical toys," he explained that all curves are compounded of two or more similar portions, which he called *integrants* of the curve; for instance, the ellipse consists of four similar quarters; the oval or egg-shape has two similar halves; the parabola and hyperbola have each two symmetric branches; the cardioid two similar halves; the two-looped, three-looped, four-looped, &c. bicircloids have four, six, eight, &c. integrants respectively; while the circle (alone) may be divided into any number of similar and equal arcs. This circumstance had enabled him to classify and tabulate the bicircloids comprised within certain limitations as to number of *loops* and *spires*, whereby he derived the following results, viz.:

Bicircloids, of which the loops or the spires									
are not more than
								3 =	10
.	9 =	70
.	99 =	7,510
.	999 =	759,480
.	5000 =	18,998,410
each of which is susceptible of innumerable <i>phases</i> , or variations of form.									

Finally, he exhibited and explained one of his *Kinegraphs*, which generates *retrogressive* or *recurrent* curves by compound circular motions. In this case, the tracing-point advanced and receded continuously in a single line *terminating in points* while delineating a *finite* curve, having for its equation

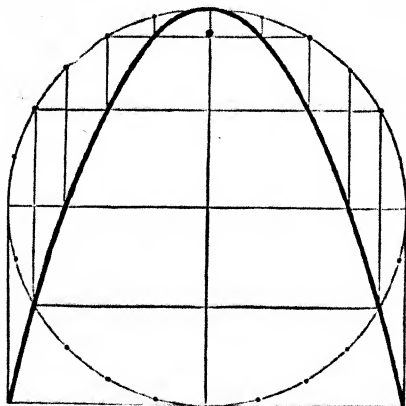
$$y = a \cos \phi, \quad x = b \cos 2\phi;$$

this, which gives $a^2 (b \pm x) = 2b y^2$,

will be recognized as *the common conical PARABOLA* within the *finite* limits of $y = \pm a$, $x = \pm b$!

Mr. Perigal, in the course of his remarks, distributed among the members and visitors various specimens of *kinematic curves* produced by *Ibbetson's Geometric Chuck*, and by other instruments adopted or contrived by himself for the elucidation of *KINEMATICS*, the science which treats of the *effects of motion*, the *causes of motion* being the province of *DYNAMICS*. He likewise distributed geometrical diagrams of the *retrogressive PARABOLA derived from the circle*, of which the following is a reprint from his stereotyped plate.

THE RETROGRESSIVE PARABOLA,
DERIVED FROM THE CIRCLE.



DISCOVERED BY PERIGAL IN 1835.

The origin of Co-ordinates at center of Circle, Radius R .

$$y = R \cos \phi; \quad x = R \cos 2\phi = R(2\cos^2\phi - 1);$$

$$\therefore 2y^2 = R(R + x).$$

y , the distance of any point of the Parabola from its axis.

The *Kinematic Curve*, of which the *Retrogressive Parabola* is a *limit*, was discovered by Mr. Perigal in 1835, and produced from *continuous motion* by him in 1840.—See *Transactions of the Society of Arts*, vol. 57. *Proceedings at Meeting of 26th March, 1851.*

[Reprinted for H. PERIGAL, 9 North Crescent, W.C. 1881.]

